# <u>SYLLABUS</u>

**Subject: MATHEMATICAL SCIENCE**

**Note:**

**There are two Papers for each of the subjects. Paper – I on Teaching and Research aptitude, Paper – II based on the syllabus of concerned subjects. Details are furnished below:**

## PAPER – I

**Subject : General Paper on Teaching & Research Aptitude**

The Test is intended to assess the teaching/research aptitude of the candidate. They are supposed to possess and exhibit cognitive abilities like comprehension, analysis, evaluation, understanding the structure of arguments, evaluating and distinguishing deductive and inductive reasoning, weighing the evidence with special reference to analogical arguments and inductive generalization, evaluating, classification and definition, avoiding logical inconsistency rising out of failure to see logical relevance due to ambiguity and vagueness in language. The candidates are also supposed to have a general acquaintance with the nature of a concept, meaning and criteria of truth, and the source of knowledge. There will be 50 questions for Paper – I.

1. The Test will be conducted in objective mode. The Test will consist of two Papers. All the two Papers will consists of only objective type questions and will be held on the day of Test in two separate sessions as under :

| Session | Paper | Number of Questions | Marks | Duration |
|---|---|---|---|---|
| First | I | 50 question | 50 ×2 = 100 | 1 Hours |
| Second | II | 100 questions | 100×2 =200 | 2 Hours |

2. Candidates who appear in two Papers and secure at least 40% aggregate marks for candidates belonging to General Category and at least 35% aggregate marks for candidates belonging to reserved categories will be declared qualifies for Eligibility for Assistant Professor by following the reservation policy of the State Government.
3. The Syllabus of Paper – II and Paper – III will be combined for Paper – II of each subject.

# MATHEMATICAL SCIENCE

## SECTION - B

## PAPER-II

**General Information :** Units 1, 2, 3 and 4 are compulsory for all candidates. Candidates with Mathematics background my omit units 10 -14 and units 17, 18. Candidates with Statistics background may omit units 6, 7 and 9. Adequate alternatives would be given for candidates with O. R. background.

1. Basic concepts of Real and Complex analysis-sequences and series, continuity, uniform continuity, Differentiability, Mean Value Theorem, sequences and series of functions, uniform convergence, Riemann integral-definition and simple properties. Algebra of Complex numbers, Analytic functions, Cauchy's Theorem and integral formula, Power series, Taylor's and Laurent's series, Residues, contour integration.

2. Basic Concepts of Linear Algebra-Space of n-vectors, Linear dependence, Basic, Linear transformation, Algebra of matrices, Rank of a matrix, Determinants, Linear equations, Quadratic forms, Characteristic roots and vectors.

3. Basic concepts of probability-Sample space, discrete probability, simple theorem on probability, independence of events, Bayes Theorem. Discrete and continuous random variables, Binomial, Poisson and Normal distributions; Expectation and moments, independence of random variables, Chebyshev's inequality.

4. Linear Programming Basic Concepts-Convex sets, Linear Programming Problem (LPP). Examples of LPP. Huperplane, Open and closed half-spaces. Feasible, basic feasible and optimal solutions. Extreme point and graphical method.

5. Real Analysis-finite, countable and uncountable sets, Bounded and unbounded sets, Archimedean property, ordered field, completeness of R, Extended real number system, lumsup and limit of a sequence, the epsilon-delta definition of continuity and convergence, the algebra of continuous functions, monotonic functions, types of discontinuities, Infinite limits and limits at infinity, func-



tions of bounded variation, elements of metric spaces.

6. Complex Analysis-Riemann Sphere and Stereographic projection. Lines, Circles crossratio. Mobius transformations, Analytic functions, Cauchy-Riemann equations, line integrals, Cauchy's theorem, Morera's theorem, Liouville's theorem, integral formula, zero-sets of analytic functions, exponential, sine and cosine functions, Power series representation, Classification of singularities, Conformal Mapping.

7. Algebra-Group, subgroups, Normal subgroups, Quotient Groups, Homomorphisms, Cyclic Groups, Permutation Groups, Cayley's Theorem, Rings, Ideals, integral Domains, Fields, Polynomial Rings.

8. Linear-Algebra-Vector spaces, subspaces, quotient spaces, Linear independence, Bases, Dimension. The algebra of linear Transformations, kernel, range, isomorphism, Matrix Representation of a linear transformation, change of bases, Linear functionals, dual space, projection, determinant function, eigen values and eigen vectors, Cayley-Hamittion Theorem, invarient Sub-spaces, Canonical Forms; diagonal form, Triangular form, Jordan form, inner product spaces.

9. Differential Equations-First order ODE, singular solutions, initial value Problems of First Order ODE, General theory of homogenous and non-homogeneous Linear ODE, Variation of Parameters, Lagrange's and Charpit's methods of solving First order Partel Differential Equations. PDE's of higher order with constant coefficients.

10. Data Analysis Basic Concepts-Graphical representation, measures of central tendency and dispersion. Bivariate data, correlation and regression, Least squares-polynomial regression, Application of normal distribution.

11. Probability - Axiomatic definition of probability. Random variables and distribution functions (univeriate and multivariate); expectation and moments; independent events and independent random variables; Bayes' theorem; marginal and conditional distribution in the multivariate case, coveriance matrix and correlation coefficients (product moment, partial and multiple), regression.

Moment generating functions; characteristic functions; probability inequalities (Tchebyshef, Markov, Jenson). Convergence in probability and in distribution; weak law of large numbers and central limit theorem for independent identically distributed random variables with finite variance.

12. Probability Distribution-Berhount, Binomial, Multinomial, Hypergeomatric, Poisson, Geometric and Negative binomial distribution, Uniform, exponential, Cauchy, Beta, Gamma, and normal (univariate and multivariate) distributions Transformations of random variables; sampling distributions, t, F and chi-square distributions as sampling distributions, Standard errors and large sample distributions. Distribution of order statistics and range.

13. Theory of Statistics : Methods of estimation : maximum likelihood method, method of moments, minimum chi-square method, least-squares method. Unblasedness, efficiency, consistency. Cramer-Rao linequality. Sufficient Stastics. Rao-Blackwell Theorem. Uniformly minimum variance unblased estimators. Estimation by confidence intervals. Tests of hypothesis : Simple and composite hypotheses, two types of errors, critical region, randomized test, power function, most powerful and unifirmly most powerful tests, Likelihood-ratio tests. Wald's sequential probability ratio test.

14. Stastical methods and Data Anlysis- Tests for mean and variance in the normal distribution : one-population and two-population cases; related confidence intervals. Tests for product moment, partial and multiple correlation coefficients; comparison of k linear regressions. Fitting polynomial regression; related test. Analysis of discrete data : chi-square test of goodness of fit, contingency tables. Analysis of varience : one-way and two-way classification (equal number of observations per cell). Large-sample tests through normal approximation.

    Nonparametric tests : sign test, median test, Mann-Whitney test, Wilcoxom test for one and two-samples, rank correlation and test of independence.

15. Operational Research Modelling - Definition and scope of Operational Research. Different types of models. Replacement models and sequencing theory, inventory problems and their analytical structure. Simple deterministic of queueing system, different per-



formance measures. Steady state solution of Markovian queueing models : M/M/1, M/M/1 with limited waiting space M/M/C, M/M/C with limited waiting space.

16. Linear Programming - Linear Programming. Simplex method, Duality in linear programming. Transformation and assignment problems. Two person-zero sum games. Equivalence of rectangular game and linear programming.

17. Finite Population : Sampling Techniques and Estimation : Simple random sampling with and without replacement. Stratified sampling; allocation problem; systematic sampling. Two stage sampling. Related estimation problems in the above cases.

18. Design of Experiments : Basic principles of experimental design. Randomisation structure and analysis of completely randomised, randomised blocks and Latinsquare designs. Factorial experiments. Analysis of 2n factorial experiments in randomised blocks.

## PAPER-III

1. Real Analysis : Riemann integrable functions; Improper integrals, their convergence and uniform convergence. Euclidean space R'', Bolzano-Welerstrass theorem, compact Subsets of R'', Heine-Borel theorem, Fourier series.

   Continuity of functions of R'', Differentiability of F:R''>Rm, Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series. Inverse function theorem, implict function theorem.

   Integral functions, line and surface integrants, Green's theorem, Stoke's theorem.

2. Complex Analysis : Cauchy's theorem for convex regions, Power series representation of Analysis function. Liouville's theorem, Fundamental theorem of algebra, Riemann's theorem on removable singularaties, maximum modulus principle, Schwarz lemma, Open Mapping theorem, Casoratti-Welerstrass-theorem, Welerstrass's theorem on uniform convergence on compact sets, Bilinear transformations, Multivalued Analytic Functions, Rimann Surfaces.

3. Algebra : Symmetric groups, alternating groups, Simple groups, Rings, Maximal ideals, Prime Ideals, Integral domains, Euclidean domains, principal Ideal domains, Unique Factorisation domains, quotient fields, Finite fields, Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner product Spaces, Orthogonality, quadratic Forms, Reduction of quadration forms. Reduction of Quadritic forms.

4. Advance Analysis : Element of Metric Spaces Convergence, continuity, compactness, Connectedness, Weierstrass's approximation Theorem, Completeness, Bare category theorem, Labesgue measure, Labesgue integral, Differentiation and Integration.

5. Advanced Algebra : Conjugate elements and class equations of finite groups, Sylow theorem, solvable groups, Jordan Holder Theorem Direct Products, Structure Theorem for finita abellean groups, Chain conditions on Rings; Characteristic of Field, Field extensions, Elements of Galois theory, solvability by Radicals, Ruler and compass construction.

6. Functional Analysis : Banach Spaces, Hahn-Banch Theorem, Open mapping and closed Graph Theorems. Principle of Uniform boundedness, Boundedness and continuity of Linear Transformations. Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthonormal Basis, Riesz-representation theorem, Bessel's Inequality, persaval's Identity, self adjoined operators, Normal Operators.

7. Topology : Elements of Topological Spaces, Continuity, Convergence, Homeomorphism, Compactness, Connectedness, Separation Axioms, First and Second Countability, Separability, Subspaces, Product Spaces, quotient spaces, Subspaces, Product Spaces, quotient spaces. Tychonoff's Theorem, Urysohn's Metrization theorem, Homotopy and Fundamental Group.

8. Discrete Mathematics : Partially ordered sets, Latices, Complete Latices, Distributive latices, Complements, Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees, Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.



9. Ordinary and Partial Differential Equations : Existence and Uniqueness of solution dyxdx-f(x,y) Green's function, sturm Liouville Boundary Value Problems, Cauchy Problems and Characteristics, Classification of Second Order PDE, Seperation of Variables for heat equation, wave equation and Laplace equation, Special functions.

10. Number Theory : Divisibility : Linear diophantine equations. Congruences. Quadratic residues; Sums of two squares, Arithmatic functions Mu, Tau, Phi and Sigma (and).

11. Mechanics : Generalised coordinates; Lagranges equation; Hamilton's coronics equations; Variational principles least action; Two dimensional motion of rigid bodies; Euler's dynamical equations for the motion of rigid body; Motion of a rigid body about an axis; Motion about revolving axes.

12. Elasticity : Analysis of strain and stress, strain and stress tensors; Geomatrical representation; Compatibility conditions; Strain energy function; Constitutive relations; Elastic solids Hookes law; Saint-Venant's principle, Equations of equilibrium; Plane problems-Airy's stress function, vibrations of elastic, cylindrical and spherical media.

13. Fluid Mechanics : Equation of continuity in fluid motion; Euler's equations of motion for perfect fluids; Two dimensional motion complex potential; Motion of sphere in perfect liquid and motion of liquid past a sphere; vorticity; Navier-Stokes's equations for viscous flows-some exatct solutions.

14. Differential Geometry : Space curves - their curvature and torsion; Serret Frehat Formula; Fundamental theorem of space curves; Curves on surfaces; First and second fundamental form; Gaussian curvatures; Principal directions and principal curvatures; Goedesics, Fundamental equations of surface theory.

15. Calculus of Variations : Linear functionals, minimal functional theorem, general variation of a functional, Euler - Larange equation; Variational methods of boundary value problems in ordinary and partial differential equations.

16. Linear integral Equations : Linear integral Equations of the first and second kind of Fredholm and Volterra type; soluting by suc-

cessive substitutions and successive approximations; Solution of equations with seperable kernels; The Fredholm Alternative; Holbert-Schmidt theory for symmetric kernels.

17. Numerical analysis : Finite differences, interpolation; Numerical solution of algebric equation; Iteration; Newton-Rephson method; Solutions on linear system; Direct method; Gauss elimination method; Matrix-Inversion, elgenvalue problems; Numerical differentiation and integration. Numerical solution of ordinary differential equation, iteration method, Picard's method, Euler's method and improved Euler's method.

18. Integral Transformal place transform : Transform of elementary functions, Transform of Derivatives, Inverse Transform, Convolution Theorem, Application, Ordinary and Partial differential equations; Fourier transforms; sine and cosine transform, Inverse Fourier Transform, Application to ordinary and partial differential equations.

19. Mathematical Programming Revised simplex method. Dual simplex method, Sensitivity analysis and perametric linear programming. Kuhn-Tucker conditions of optimality. Quadratic programming; methods due to Beale, Wofle and Vandepanne, Duality in quadratic programming, self duality, Integer programming.

20. Measure Theory : Measurable and measure species; Extension of measure, signed measure, Jordan-Hahn decomposition theorems. Integration, monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Absolute continuity. Radon Niiodymtheorem, Product measures, Fubinl's theorem.

21. Probability : Sequences of events and random variables; Zero-one laws of Borel and Kolmogorov. Almost sur convergence, convergence in mean square, Khintchine's weak law of large numbers; Kologorov's inequality, strong law of large numbers. Convergence of series of random variables, three-series criterion. Central limit theorems of Liapounov and Lindeberg-Feller. Conditional expectation, martingales.

22. Distribution Theory : Properties of distribution functions and characteristic functions; continuity theorem, inversion formula, Representation of distribution function as a mixture of discrete and



continuous distribution functions, Convolutions, marginal and conditional distributions of bivariate discrete and continuous distributions.

Relations between characteristic functions and moments: Moment inequalities of Holder and Minkowski.

23. Statistical inference and Decision Theory : Statistical Decision problem; non-randomized, mixed and randomized decision rules; risk function, admissibility, Bayes' rules, minimax rules, least favourable distributions, complete class and minimal complete class. Decision problem for finite perameter space. Convex loss function. Role of sufficiency.

Families of distributions with monotone likelihood property, exponential family of distributions. Test of simple hypothesis against a simple alternative from decision, theoretic viewpoint. Tests with Neyman structure. Uniformly most powerful unbiased tests. Locally most powerful tests, inference on location and scale parameters; estimation and tests. Equivariant estimators, invarience in hypothesis testing.

24. Large sample statistical methods : Various modes of convergence. Op and op, CLT, Sheffe's theorem, Polya's theorem and Slutsky's theorem, Transformation and varience stabilizing formula. asymptotic distribution of function of sample moments, Sample quantiles, Order statistics and their functions, Tests on correlations, coefficients of variation, skewness and kurtosis, Pearson Chi-square, contingency Chi-square and childhood ratio statistics, U-statistics, Consistency of Tests, Asymptotic relative efficiency.

25. Multivariate statistical Analysis : Singular and non-singular multivariate distributions. Characteristics functions Multivariate normal distribution; marginal and conditional distribution, distribution of linear forms, and quadratic forms, Cochran's theorem.

inference on parameters of multivariate normal distributions; one population and two-population cases. Wishart distribution, Handlings T2, Mahalenobils D2, Discrimina-Analysis, Principal components, Canonical correlations, Cluster analysis.

26. Linear Models and Regression : Standard Gauss Markov models; Estimability of parameters; best linear, unbised estimates (Bell..);

Method of least squares and Gauss-Markov theorem; Variance-coveriance matrix of BLUES. Test of linear hypothesis, One-way and two-way classifications. Fixed, random and mixed effects models (two-way classifications only); variance components, Bi-variable and multiple linear regression; Poly-normal regression; use of orthogonal poly-normals. Analysis of coveriance. Linear and nonlear regression, Outliers.

27. Sample Surveys : Sampling with varying probability of selection, Hurwitz-Thompson estimator; PPS sampling; Double sampling. Cluster sampling. Non-sampling errors : Interpenetrating samples. Multiphase sampling. Ratio and regression methods of estimation.

28. Design of Experiments : Factorial experiments, confounding and fractional replication. Split and strip plot designs; Quasi-Latin square designs; Youden square. Design for study of response surfaces; first and second order designs. Incomplete block designs; Balanced, connectedness and orthogonality, BIBD with recovery of inter-block information; PBIBD with 2 associate classes. Analysis of series of experiments, estimation of residual effects. Construction of orthogonal-Latin squares, BIB designs and confounded factorial designs. Optimality criteria for experimental designs.

29. Time-Series Analysis : Discrete-parameter stochastic processes; strong and weak stationary; autocovariance and autocorrelation. Moving average, autoregressive, autoregressive moving average and autoregressive integrated moving average processes. Box-Jenkins models. Estimation of the parameters in ARIMA models; forecasting. Periodogram and correlogram analysis.

30. Stochastic Process : Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution; branching processes; Random walk; Gambler’s ruin. Markov processes in continuous time; Poisson processes, birth and death processes, Wiener process.

31. Demography and Vital Statistics : Measures of fertility and mortality, period and Cohort measures. Life tables and its applications; Methods of construction of abridged life tables. Application of stable population theory to estimate vital rates. Population projections. Stochastic models of fertility and reproduction.



32. Industrial Statistics : Control charts for variables and attributes; Acceptance sampling by attributes; single, double and sequential sampling plans; OC and ASN functions, AOQL and ATI; Acceptance sampling by varieties. Tolerance limit. Reliability analysis : Hazard function, distribution with DFR and IFR; Series and parallel systems. Life testing experiments.

33. Inventory and Queueing theory : Inventory (S,s) policy, periodic review models with stochesitic demand. Dynamic inventory models. Probabilistics re-order point, lox size inventory system with and without lead time. Distribution free analysis. Solution of inventory problem with unknown density function. Warehousing problem. Queues : Imbeded markov chain method to obtain steady state solution of M/G/1,G/M/1 and M/D/C, Network models. Machine maintenance models. Design and control of queing systems.

34. Dynamic Programming and Marketing : Nature of dynamic programming, Determinstic processes, Non-sequential discrete optimisation-allocation problems, assortment problems. Sequential discrete optimisation long-term planning problems, multistage production processes. Functional approximations. Marketing systems, application of dynamic programming to marketing problems. Introduction of new product, objective in setting market price and its policies, purchasing under fluctuating prices. Advertising and promotional decisions. Brands switching analysis, Distribution, decisions.